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Fractional statistics and confinement

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It is shown that a pointlike composite having charge and magnetic moment displays a confining potential for the static interaction while simultaneously obeying fractional statistics in a pure gauge theory in three dimensions, without a Chern-Simons term. This result is distinct from the Maxwell-Chern-Simons theory that shows a screening nature for the potential.

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The explanation for the fact that the fundamental constituents of matter are confined is a long-standing theoretical physics problem. Its solution has evaded full comprehension up to now and a great many deal of progress has been made in order to distinguish between the apparently related phenomena of screening and confinement. In fact the distinction between screening and confinement is of considerable importance in our present understanding of gauge theories. In order to avoid the complexities of four dimensions the strategy of many of these studies has been the restriction to lower dimensions as a theoretical laboratory.

In this work we show that a pointlike composite having charge and magnetic-dipole is able to provide a clear-cut realization of anyon fractional statistics in (2+1) dimensions while displaying a confining feature among two such composites. The anyonic fractional statistics [1] displayed by this system is realized without the introduction of a Chern-Simons term, this way avoiding the appearance of topological mass that is responsible for the screening characteristics displayed by pure electric charges in the electrodynamics controlled by the Maxwell-Chern-Simons theory [2]. Differently, the electrodynamics of the composite studied here is a pure Maxwell theory. In this case a rich physical structure was found to exist and the issue of confinement has been satisfactorily settled. The full understanding is obtained by studying the potential created by two oppositly charged composite some distance apart.

Electrodynamics in (2+1) dimensions have been extensively discussed in the last few years[3, 4, 5]. It is well known that in three dimensions it is allowed the possibility of particles with any statistics, where the physical excitations obeying it are called anyons. A concrete way to realize non-trivial statistics is by attaching a magnetic flux to electrically charge particles. A simple way to attach a flux to an elementary charge is to use the Chern-Simons theory to control the dynamics of the photons so that Wilczek's charge-flux composite model of the anyon can be implemented[1].

However, it should be recalled that when the Chern-Simons term is added to the usual Maxwell term, the gauge field becomes massive, leading to topologically massive gauge theories. Consequently electric and magnetic fields are virtually screened at distances bigger than $\Lambda \approx 1/m$ and the statistics of anyons changes with their mutual distance in the presence of the Maxwell term [6]. However, it has been shown possible to describe anyons without the introduction of Chern-Simons term[7, 8] through the introduction of a generalized connection allowing this way the realization of fractional statistics.

In the present context it is also important to recall that the interaction potential between static charges is an object of considerable interest. In fact the ideas of screening and confinement play a central role in modern gauge theory and the physical content can be understood when a correct separation of the physical degrees of freedom is made [9] using a formalism in terms of physical (gauge-invariant) quantities. This method has been used previously for studying features of screening and confinement in two-dimensional quantum electrodynamics, generalized Maxwell-Chern-Simons gauge theory and for the Yang-Mills field [9]. After the dynamical feature of the composite is presented below, this method will be applied to study the interaction energy of two such entities.

As already stated, our objective is to compute explicitly the interaction energy between static pointlike sources for this new electrodynamics. For this purpose we shall first carry out its Hamiltonian analysis. The gauge theory we are considering is defined by the following Lagrangian in three-dimensional space-time:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - A_\mu J^\mu. \tag{1}$$

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For a static composite locate at $\mathbf{x_0}$, the "charge-magnetic dipole" is defined by $J_0 = \rho$ and $J_k = \frac{\mu}{e}\epsilon_{km}\partial_m\rho$ and has been introduced before by Deser [10] in another context, where $\rho = e\delta^{(2)}(\mathbf{x}-\mathbf{x_0})$ and μ is the dipole's moment [11]. For a composite located at the origin, the solution of the Maxwell equations read,

$$B(\mathbf{x}) = \mu\delta^{(2)}(\mathbf{x}), \tag{2}$$

$$E_i(\mathbf{x}) = -\frac{e}{2\pi}\frac{x^i}{r^2}, \tag{3}$$

with $r \equiv |\mathbf{x}|$. One immediately sees that the associated magnetic field has its support only at the position of the composite, and a long range electric field is also generated. As a consequence, the total magnetic flux associated to the magnetic field is

$$\Phi_B = \int\limits_V d^2x B(x) = \mu. \tag{4}$$

This tells us that the charged dipole actually behaves like a magnetic flux point. As it turns out the composite can realize fractional statistics without introducing a Chern-Simons term. This provides a novel way to describe anyons. Accordingly, the mechanism of attaching a magnetic flux to the charges has been implemented in a particularly simple way, as was claimed in Ref. [7].

Next let us consider the screening versus confinement question. It is well known that under the Maxwell-Chern-Simons dynamics, the static potential for two oppositely charged particles is given by

$$V = -\frac{e^2}{\pi}K_0(\eta|\mathbf{y}-\mathbf{y}'|), \tag{5}$$

where K_0 is a modified Bessel function and η is a massive cutoff. This result displays the screening character of the MCS potential for anyons.

To obtain the corresponding behavior for the charge-magnetic dipole composite we must carry out the quantization of the system. The Hamiltonian analysis of the system starts with the computation of the canonical momenta $\Pi^\mu = F^{\mu 0}$, which produces the usual primary constraint $\Pi^0 = 0$ and $\Pi^i = F^{i0}$ ($i = 1, 2$) while the canonical Hamiltonian has the usual maxwellian structure

$$H_C = \int d^2x \left\{ -\frac{1}{2}F_{i0}F^{i0} + \frac{1}{4}F_{ij}F^{ij} - A_0\left(\partial_i\Pi^i - J^0\right) + A_iJ^i \right\}. \tag{6}$$

Preservation in time of the primary constraint leads to the secondary constraint $\Omega_1(x) \equiv \partial_i\Pi^i(x) - J^0(x) = 0$ and together displays the first-class structure of the theory. Using $c_0(x)$ and $c_1(x)$ as Lagrange multiplier fields to implement the constraints, the corresponding total (first class) Hamiltonian that generates the time evolution of the dynamical variables then reads

$$\begin{aligned} H &= H_C + \int d^2x \left\{c_0(x)\Pi_0(x) + c_1(x)\Omega_1(x)\right\} \\ &= \int d^2x \left\{ -\frac{1}{2}F_{i0}F^{i0} + \frac{1}{4}F_{ij}F^{ij} + c'(x)\left(\partial_i\Pi^i - J^0\right) + A_iJ^i \right\} \end{aligned} \tag{7}$$

where $c'(x) = c_1(x) - A_0(x)$. We have used that $\Pi^0 = 0$ for all time and $\dot{A}_0(x) = [A_0(x), H] = c_0(x)$, which is completely arbitrary, to eliminate A^0 and Π^0.

To fix the gauge it is convenient to choose

$$\Omega_2(x) \equiv \int\limits_{C_{\xi x}} dz^\nu A_\nu(z) = \int_0^1 d\lambda x^i A_i(\lambda x) = 0 \tag{8}$$

as a constraint, where λ $(0 \le \lambda \le 1)$ is the parameter describing the spacelike straight path between the reference point ξ^k and x^k , on a fixed time slice. For simplicity we have assumed the reference point $\xi^k = 0$. The choice (8) leads to the Poincaré gauge [9]. Through this procedure, we arrive at the following set of Dirac brackets for the canonical variables

$$\left\{A_i(x), A^j(y)\right\}^* = 0, \tag{9}$$

$$\left\{\pi_i(x), \pi^j(y)\right\}^* = 0, \tag{10}$$

$$\left\{A_i(x), \pi^j(y)\right\}^* = g_i^j \delta^{(2)}(x-y) - \partial_i^x \int_0^1 d\lambda \; x^j \delta^{(2)}(\lambda x - y), \tag{11}$$

while the Dirac brackets of the magnetic $\left(B = \varepsilon_{ij}\partial^i A^j\right)$ and electric $(E^i = \Pi^i)$ fields are the canonical ones:

$$\{E_i(x), E_j(y)\}^* = 0, \tag{12}$$

$$\{B(x), B(y)\}^* = 0, \tag{13}$$

$$\{E_i(x), B(y)\}^* = -\varepsilon_{ij}\partial_x^j \delta^{(2)}(x-y). \tag{14}$$

It is important to realize that, unlike the Maxwell-Chern-Simons theory, in the present model, the brackets (9) and (12) are commutative. The equations of motion for the electric and magnetic fields are,

$$\dot{E}_i(x) = -\varepsilon_{ij}\partial^j B(x) + J_i(x) + \int d^2y J^k(y)\,\partial_k \int_0^1 d\lambda y_i \delta^{(2)}(\lambda x - y), \tag{15}$$

$$\dot{B}(x) = -\varepsilon_{ij}\partial_i E_j(x). \tag{16}$$

In the same way, we write the Gauss law as:

$$\partial_i E_L^i - J^0 = 0, \tag{17}$$

where E_L^i refers to the longitudinal part of E^i. These are the Ampere, Faraday and Gauss laws, respectively. Together they imply that for a static composite located at $x^i = 0$, the static electromagnetic fields are given by Eqs. (2) and (3). The set of Dirac brackets above is then elevated to the category quantum commutators as usual [12].

After achieving the quantization we may now proceed to calculate the interaction energy between pointlike sources in the model under consideration. To do this, we will compute the expectation value of the energy operator H in a physical state $|\Omega\rangle$. We also recall that the physical states $|\Omega\rangle$ are gauge-invariant [13]. In that case we consider the stringy gauge-invariant $\left|\overline{\Psi}(\mathbf{y})\,\Psi(\mathbf{y}')\right\rangle$ state,

$$|\Omega\rangle \equiv \left|\overline{\Psi}(\mathbf{y})\,\Psi(\mathbf{y}')\right\rangle = \overline{\psi}(\mathbf{y}) \exp\left(-ie\int_{\mathbf{y}}^{\mathbf{y}'} dz^i A_i(z)\right)\psi(\mathbf{y}')\,|0\rangle, \tag{18}$$

where $|0\rangle$ is the physical vacuum state and the integral is to be over the linear spacelike path starting at $\mathbf{y}$ and ending at $\mathbf{y}'$, on a fixed time slice. Note that the strings between fermions have been introduced to have a gauge-invariant state $|\Omega\rangle$, in other terms, this means that the fermions are now dressed by a cloud of gauge fields. We can write the expectation value of H as

$$\langle H\rangle_\Omega = \langle\Omega| \int d^2x \left(\frac{1}{2}E_i^2 + \frac{1}{2}B^2 + J^i A_i\right)|\Omega\rangle. \tag{19}$$

The preceding Hamiltonian structure thus leads to the following result

$$\langle H\rangle_\Omega = \langle H\rangle_0 + \frac{e^2}{2}\int d^2x \left(\int_{\mathbf{y}}^{\mathbf{y}'} dz_i \delta^{(2)}(x-z)\right)^2, \tag{20}$$

where $\langle H\rangle_0 = \langle 0|\,H\,|0\rangle$. Following our earlier procedure [14], we see that the second term on the right-hand side of Eq. (20) is clearly dependent on the distance, and the potential for two opposite located at $\mathbf{y}$ and $\mathbf{y'}$ takes the form

$$V = \frac{e^2}{\pi}\ln\left(\eta|\mathbf{y} - \mathbf{y}'|\right). \tag{21}$$

This result displays the confining character of the Maxwell potential for the composites. We see that the result (21) agrees with the behavior of the Maxwell-Chern-Simons theory, Eq.(5), in the limit of short separation.

In summary we have shown that while both systems displays fractional statistics, the composite system proposed here interacts with purely maxwellian photons and displays a potential with the confining nature (the potential grows to infinity when the mutual separation grows). Differently, it is known that use of the Chern-Simons term turns the electric and magnetic fields massive leading to a screening potential between static charges. The analysis above has clearly revealed that, although both theories lead to fractional statistics by the same mechanism of attaching a magnetic flux to the charges, their physical contents are quite different. The observation in the present work that the composite leads to fractional statistics and confinement is new.

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